The computational estimation and instructional perspectives of elementary school teachers

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ABSTRACT

The purpose of this study is to investigate teachers' understanding and knowledge of computational estimation, and teaching practice toward to computational estimation. There are six fifth-grade elementary teachers were participated this study; three teachers with mathematics/ science major and three teachers with non-mathematics/science major. Each teacher received the interviews for three times and the interview time was about 90 minutes. The major findings are as follows: In the teacher's knowledge about computational estimation aspect, six teachers could explain the meaning of computational estimation, and use computational estimation strategy to solve problems flexibly. Their computational estimation strategies to solve problems include front-end, rounding, compatible number, special number, use of fractions, nice number and distributive property. All of teachers use special numbers; five of them use rounding and compatible number strategies. Four teachers use nice numbers. Only one teacher use front-end strategy and distributive property. The results indicate that the teachers with mathematics/science major were more abundant of the knowledge about computational estimation relatively. The teachers are non- mathematics/ science majors but were extremely interested in mathematics had initial understanding toward the related knowledge of computational estimation. All these teachers incorporated mathematics course with computational estimation, or integrated with other mathematics concepts, required students to examine the answer and its rationality by computational estimation, introduce the life situation with teaching to improve student's learning motivation. However; the teachers are non-mathematics /science major and not interested mathematics has less computational estimation strategy for their teaching.

Keywords: computational estimation, computational estimation strategy

INTRODUCTION

An overview of mathematics curriculum standards in Taiwan and international shows that more countries have come to stress computational estimation instruction, emphasizing its importance (Australian Education Council, 1991; National Council of Teachers of Mathematics, 2000; Ministry of Education, 2003). The National Council of Teachers of Mathematics (2000) published the "Curriculum and Evaluation Standards for School Mathematics," in which "numbers and computation" suggests that students should be able to fluently compute and make reasonable computational estimations; in addition to the continued emphasis on the importance of computational estimation, it also suggested the enhancement of computational estimation instruction. Trafton (1986) stressed that "building a strong computational estimation strand into school mathematics programs must be a top priority for curriculum developers" (p. 16).

In past mathematical instruction, computational estimation is an issue that has been overlooked. As the examination system has guided instruction, in order to obtain higher mathematics scores, education has tended to emphasize pen-and-paper computations, speed, and a single answer. There has always been emphasis on drilling practice and familiarity with calculation capabilities of the computational principles, while there is a lack of cultivation of thinking and logical deduction (Yang, 2000, 2001). However, with the continuous reform of education, currently curricular standards have also placed increasing emphasis on the field of computational estimation, believing that learning computational estimation could significouldtly help students' number sense and mathematical thinking.

Estimation is not only a valuable skill in its own right, it can also be a valuable pedagogical tool used in the development of other important skills (Buchanan,1978). Students with better number sense could generally use computational estimation techniques to assist in computations, checking, and solutions(Tsao, 2009; Tsao & Pan, 2010) . The instruction of computational estimation problems could promote students to further understand the connections among mathematics concepts, equation computations, and problem-solving (Ministry of Education, 2003). Thus, "computational estimation" in one of the five major themes in Taiwan's nine-year uniform mathematics curriculum, "numbers and quantities" emphasizes the usage of concrete contexts for computational estimation, as well as the process of using concrete contexts to estimate and explain the process of computational estimation. This shows that the sector of mathematics education has gradually emphasized the cultivation of computational estimation ability, and computational estimation instruction has also become an important direction for reform.

The significance of computational estimation and its place in the learning and using of mathematics has been highlighted by many respected mathematics educators. It is clearly understand the importance of applying computational estimation in daily life, thus it is very important to parepae students to have computational estimation skills. Then students would come to view mathematics as a distinct way of thinking, rather than as a collection of unconnected rules.

In the meanwhile, computational estimation instruction has received the emphasis of many advanced nations. Even though the mathematics curriculum in Taiwan includes the content of computational estimation, but there should be further exploration of how to incorporate the concept of computational estimation in school instruction into life, and to integrate the connection. Thus, the concept of computational estimation is not as clear as fractions and decimals, so it is worth considering whether teachers could incorporate the application of computational estimation in instruction. This study focus on "teachers" to explore the views of elementary school teachers toward their computational estimation knowledge (including computational estimation content, computational estimation instruction, and computational estimation problem-solving).

RESEARCH QUESTIONS

(1) What is the understanding of elementary school teachers with respect to the meaning and importance of computational estimation?
(2 What computational estimation strategies do elementary teachers apply when asked to teach involving computational estimation?
(3)What is the relation between computational estimation instruction of elementary school teachers and their backgrounds?
(4) What computational estimation strategies do elementary school teachers use when asked to solve problems involving computational estimation?

COMPUTATIONAL ESTIMATION AND COMPUTATIONAL ESTIMATION INSTRUCTION

Computational estimation is finding an approximate answer to arithmetic problems without actually computing the exact answer. It is an important component of mathematical cognition, as it provides information about people's general understanding of mathematical concepts, relationships, and strategies, and about children's cognitive development in the domain of mathematics (Sowder, 1988, 1992; Sowder & Wheeler, 1989). Moreover, computational estimation skills are useful in everyday situations in which a rough answer provides a contextually appropriate degree of precision.

Jie (1993, 1996) conducted studies on computational estimation, including the development of elementary school children's computational estimation and teachers' computational estimation instruction in Taiwan. The study found that in the computation process, few students discover the existence of unreasonable phenomena, and the willingness to use computational estimation seems to be correlated to habit. This indicatied that students' experience in computational estimation appear insufficient, similar to Tsao and Pan (2011)'s finding that Taiwan's elementary school students knew concepts of "about", "roughly", but rarely used computational estimation. Most teachers have heard of the term "computational estimation," and the most commonly used method of "rounding strategy" in solving continuous addition problems. Half of the people also use the clustering strategy, and most teachers could determine the reasonableness of answers. Approximately 80% of the teachers believe that computational estimation is very important in daily life, and think that computational estimation content should be added in mathematics curriculum (Jie, 1993, 1996)

This results indicate that a lack of clear guidelines means teachers easily misunderstand the real objectives of estimation instruction and teachers teach their students a simple rounding strategy rather than providing them with a complete introduction to and training in various estimation strategies. Researchers indicated that teachers should employ appropriate examples to help students learn different estimation strategies and then employ problems to stimulate students to select an appropriate problem solving strategy via group discussion and reciprocal learning (Trafton, 1988; van de Walle , 2005)

Computational estimation instruction could help students learn the concept of numbers and understand the process, or that in the process they could not only learn computational estimation techniques as well place value, size of values, and the factual concepts on numbers, and the meanings of computations, further perceiving the reasonableness of answers. Meng (1998) indicated that knowledge in computational estimation instruction refers to combining computational estimation knowledge and instructional knowledge, and before the job, when teachers conduct computational estimation instruction they could intergate the content of computational estimation (e.g., computational estimation strategy, the problems in computational estimation reasonableness, etc.) and instructional knowledge (e.g., setting up problems, discussion, presentation, etc.) into computational estimation instruction. The "Developing number sense for middle graders" published by National Council of Teachers of Mathematics (NCTM, 1991), mentioned that one of the roles played by teachers is to “encourage and accept good calculation methods”: students could use pen-and-paper computations, mind computations, computational estimation, and calculators to promote development of number sense. In the instruction process, encouraging students to share strategies of mind computation and computational estimation with peers could help them understand basic relationships and computations principles of digit positions. Furthermore, under some contexts, for the understanding of some concepts, mind computation and computational estimation are more effective than precise computations. Advancement in computation must rely on continuous practice, just as some mathematical techniques could be improved only through practice. If computational estimation is concentrated in instruction in some units, it would limit computational estimation to parts of the computation system, and when students see computational estimation problems they would use rigid strategies for computation (Sowder, 1984).

In summary, computational estimation instruction could reinforce the computational estimation ability of students, but it would require continuous practice. However, when teachers teach computational estimation, they either efficiently achieve the goal by directly teaching students about strategies of computational estimation, or encourage students to explore and create by using indirect strategies. In fact, indirect instructional methods could help students develop better number concept and number sense. When students freely explore computational estimation strategies, they could use their number intuition, and sometimes could use non-regular methods to check the reasonableness of answers. Thus, in order to make students have computation consciousness and computational estimation abilities, teachers need to connect to the actual lives of students, so that they could use different computational estimation strategies based on students’ own needs, thinking, and individual differences. This allows students to understand the actual meaning of computational estimation, and could use their own computational estimation strategies to resolve daily life problems. Of course, in the whole mathematical instructional process, students should be encouraged to use computational estimation to cultivate their interest and enhance their computational estimation ability.

RESEARCH METHOD

Research design

The main purpose of this study is to understand the computational estimation knowledge and instructions of elementary school teachers. The research subjects of this study are current fifth grade mathematics teachers at elementary schools in Taipei. A total of six teachers with mathematics and science majors and non-mathematics and science majors who could accommodate the research and are willing to undergo interviews are the research subjects in in-depth research. At the beginning of the interview, the researcher initially accessed to nine teachers’ backgrounds, Preference of mathematics and number of years of teaching. Subsequently, the researcher used the research instrument to interview the teachers in order to understand their computational estimation knowledge, instructions of computational estimation instructions and problem-solving of computational estimation. Each teacher received the interviews for three times and the interview time was about 90 minutes.

Research subjects

The researcher interviews six mathematics teachers in fifth grade in elementary school, who are from mathematics and science majors and non-mathematics and science majors, and willing to accommodate the research. The names referred to in the text are codenames. Table 1 shows the majors of the teachers in this study.

Table 1 Background information of teachers participating in the study

Codes for teachers	Number of years teaching	Background	Preference for mathematics
A	7 years	Non-mathematics and science majors	Interested
B	8 years		Neutral
C	12 years		Interested
D	4 years	Mathematics and science majors	Highly interested
E	6 years		Highly interested
F	7 years		Highly interested

Instrument

The researchers developed a set of computational estimation interview questions, including the content of computational estimation, computational estimation instruction, and computational estimation problem-solving. Among them, the component of computational estimation instruction explores to understand teacher computational estimation instruction, including the computational estimation concepts in instructional materials in fifth grade in elementary school. Computational estimation problem-solving is the actual problem-solving methods when they were asked to solve problems by computational estimation strategy.

Interview questions

The content of computational estimation

1. What do you think the meaning of computational estimation is? What are its characteristics?

Teacher computational estimation instruction

Below are some of the interview questions for the teachers on computational estimation instruction:

2. When you first introduce computational estimation to students, how do you start with the topic for instruction?
3. Please describe how you carry out computational estimation instruction for the following questions when you teach elementary school children? (also, please explain the interaction between you and the students)
   (1) What is the approximate answer of 44÷0.78?
   (2) What is the approximate answer of 53687+8365+1638+28?
   (3) What is the approximate answer of $\frac{12}{13}+\frac{1}{17}+\frac{8}{15}$?
   (4) When Zhijie used a calculator to calculate 210.16×28.7, he forgot to press the decimal point, and got 21016×287=6031592, what should the correct answer be?
4. Which concepts among elementary school mathematics concepts do you think implicate computational estimation?
5. Other than instruction in computational estimation units, would you use computational estimation knowledge while teaching other mathematics courses?

Teachers' computational estimation problem-solving

6. Please explain how you yourself would solve the following problems in computational estimation outside of instruction?
   (1) Please estimate the approximate answer of 53687+8365+1638+28?
   (2) Please estimate the approximate answer of 12.6×11.4?
   (3) What is the approximate answer of 6470×8÷73?
   (4) What is the approximate answer of $\frac{16}{15}\times\frac{23}{47}$?
   (5) Today the exchange rate is 1 USD to 32.28 NTD, how much as a USD 0.75 toothbrush in NTD?

(6) When doing whole number divisions, how many digits might the answer to quaternions ÷ 2 digit number be?

Interview and data analysis

In order to understand the internal perspectives of people, researchers need to interview them to understand their beliefs, dreams, motivations, determinations, values, attitudes, and emotions (Langness & Frank, 1981). The researcher engages in one-on-one interviews with teachers regarding their computational estimation knowledge. The entire interview process was recorded, which was then converted to written data. Data analysis systematically searches and organizes the data collected in research to enhance researchers understanding and findings from the data.

RESEARCH RESULTS AND DISCUSSION

The computational estimation knowledge of teachers

This section analyzes the interview data to understand the views of elementary school teachers toward their computational estimation knowledge (including computational estimation content, computational estimation instruction, and computational estimation problem-solving).

Content of computational estimation

The teachers are asked what they think about computational estimation, to analyze their understanding for computational estimation content. The six teachers could all make different explanations for computational estimation. Three teachers (B, E, F) use definitions to explain computational estimation. Integration of their so-called computational estimation means using approximate computations to find approximate values, and the process is fast and time-saving, it simplifies computations and is easy to remember, but the error between the obtained computational estimation values and the precise values have to be reasonable.

The other two teachers (A, C) use computational estimation in explaining broad life applications, and when it is impossible to accurately grasp various precise data, it is necessary to estimate to grasp the concept of approximate numerical ranges. Of course, it does not only refer to numbers, but also include time, length, weight, and other concepts. Teacher D explains computational estimation using definitions and life applications.

Teacher D

TD: I think that the meaning of computational estimation is mostly in life! There is a lot of need for this in life, such as the annual income of a company. So maybe you know the rough range of something, and do not need to know the actual value, and this could help you remember and compute.

Table 2 shows the teachers' computational estimation content, which shows that the six teachers could all explain computational estimation in different degrees. Teachers with mathematics and science majors generally use definitions to interpret computational estimation, while teachers who do not have mathematics and science majors but are interested in mathematics would use life applications to explain the content of computational estimation. Only one teacher with mathematics and science background uses definitions and life applications to explain.

Table 2 Teachers' computational estimation content table

Computational estimation content \ Teacher code	A	B	C	D	E	F
Explained by life application	✓		✓	✓		
Explained by definition		✓		✓	✓	✓

Teachers' computational estimation instruction

Computational estimation instruction is the understanding of teachers regarding the conditions of computational estimation instruction, including the computational estimation concepts in the instructional materials in mathematics for the fifth grade in elementary school.

Teacher A

Before the unit, in order to understand the students' preexisting knowledge, Teacher A would use a preview sheet to understand whether students have complete basic ability, focus on unfamiliar basic concepts for review, before officially starting the unit. When teaching the computational estimation unit, they would use examples from life so that the children could determine the computational estimation based on the needs of the problem. Team competition or personal practice is used to clarify the mistaken concepts of students. In teaching, students are divided into groups to discuss the methods, and compare between teams, to discover the process of how to determine the best place value without great difference and could be calculated differently. These methods are used to understand the keys and techniques of computational estimation. For the same context, if there are differences in relatively large numbers, such as the interview question "What is the approximate answer of 53687＋8365＋1638＋28?" Teacher A would tell students to accommodate the overall number places, and the smallest number place is 28, in the tens, so the same place value is used to take the approximate number.

When there are no tools to use, it is possible to rely on the computational estimation function, such as obtaining answers more quickly while using four arithmetic operations or for verifying answers, to avoid impossible answers. In addition, the part for quantity is the same, including measurement of space and length, even angles and geometry in computational estimation; these are parts of computational estimation that are similar to computational estimation concepts.

Teacher B

When starting the computational estimation unit, Teacher B directly introduces examples from curricular content. She believes that before the computational estimation unit, preexisting knowledge needed by students include basic four arithmetic operations abilities and mind computation ability. After instruction, it is possible to apply computational estimation in life, and could reduce the error of computational estimation values. When carrying out computational estimation instruction, textbook content was the focal point of instruction, and the more difficult content is taught only if there is more time. The students are generally unable to obtain reasonable computational estimation values because the values are too big. After students have a basic concept, Teacher B then uses questions to make students think, to reinforce their self-confidence, and guide them so that they could find how to determine values in computational estimation to minimize the errors, making computational estimation easier.

Affected by the instructional content, Teacher B would emphasize that if there are options to choose from, there is greater clarity about how precise the children should be, and most use the rounding strategy method to change the numbers that are difficult to compute. Children are more familiar with whole number four arithmetic operations, but more adept students would suggest using more precise numerical values for computational estimation. Explanations would use comparisons of values to make students compare the sizes, for instance: “the computational estimation results are slightly smaller than the original.” But when the numerical values are close, the teacher would want to teach with precise computations.

When encountering open-ended questions, Teacher B thinks that this is more difficult for children, because textbooks on the market are generally all closed questions, with fixed answers to choose from. Students are unfamiliar with open-ended questions, and textbooks generally clearly set which methods are used for answers, for instance: using rounding up, so there are no ideas about how to teach open-ended questions. In the face of problems in life, students are permitted to determine what to keep and discard. As long as there are reasonable methods and errors, Teacher B could accept various solutions by students. Teacher B points out that the mathematical concepts that involve computational estimation in the course include four arithmetic operations and compare the sizes, but other than the computational estimation concept mentioned in the computational estimation unit, computational estimation is rarely used in other units. Sometimes the students are asked to use computational estimation to check the reasonableness of answers in a comparative standard.

Teacher C

When starting the computational estimation unit, Teacher C uses the example of number of students in the school to guide students about how to grasp numerical values. The number of people in class could clearly show the number of people, and not every value has to have an approximation. The numerous people in school could be described with estimated values, then bringing out the approximation methods in the course, including rounding down, rounding up, and rounding strategy. The requirements of the questions are used to select suitable methods. The approximate numbers are approximate, and computational estimation implicates computations. Not every numerical value is used or could be precisely presented. Thus, it is necessary to know the approximate and rough meaning. The numbers that couldnot be grasped are approximated, and then the computational estimation is used. When they encounter things that are difficult to compute, they could quickly obtain answers, and the computational estimation values are not very different from the accurate values. It is hoped that after students learn computational estimation, they could learn to apply it in their life contexts. Students generally have questions in regards to: if the question does not clearly require them to compute using certain methods at certain places, then they are unable to determine how to take the approximate value for computational estimation. Maybe the value they take are still reasonable, or students would face the problem of whether to first use computational estimation then compute, or first compute then estimate. At this time, Teacher C would let students understand that using the most simplified and fastest way for computation is the true meaning of computational estimation. Teacher C would not restrict students in using computational estimation to solve problems, if preexisting experiences are sufficient to resolve problems it is acceptable as well. In determining numbers, Teacher C would not regulate student methods; the children discuss and present the place values they are determined to take, but the teacher would also guide students to understand the size of errors. When determining numbers, they even bring out other mathematics concepts, such as the concepts of factors and multiples, so that students could understand the relationships among numbers, and the taken numbers should accommodate each other, to see whether there are suitable adjustment methods to make the computations more simplified.

Teacher C believes that in elementary school mathematics courses, quotient computational estimation is a concept that quite clearly uses computational estimation, and others are approximate values and not precise values in the four arithmetic operations. Thus, computational estimation is only mentioned in number concept and computational estimation units. Otherwise, other than division quotient computational estimation, general mathematics concepts do not mention computational estimation knowledge.

Teacher D

Teacher D believes that the traditional way of lecturing based on the textbook one problem after another would lengthen the process of student concept establishment and lower their learning efficiency, thus the instructional process first involves browsing the whole unit to find important points. In the curricular content, the most important are the three methods of rounding down, rounding up, and rounding strategy. After the teacher lists them, the teacher would pose examples from life or common sense points in life, so students could understand the meaning of these terms, then follow the example questions in the textbook, which would lead to higher learning interest and sense of accomplishment. They could also clarify concepts that they did not understand before. Other than example problems from textbooks or posed by the teacher, the teacher would also allow students to share their related experiences, pose real examples so that people could discuss them. Intangibly, this reinforces learning interest and allows children to find that mathematics could be used in daily life. Mathematics needs to be used flexibly, so it is hoped that students would know when to use which methods to get closer to the precise value. However, since they have fewer life experiences, they are unable to determine on their own which methods to use at what times.

Teacher D believes that in instruction, the children should not be told how to solve problems. This is because traditional instruction has already deprived students of creative thinking, and it is necessary to let them develop the methods they want by encouraging different ideas to discover faster methods, which would then become their own. Thus, Teacher D would first ask the students about the methods they use to see how much they understand and to provide suitable guidance. Other than the regular computational estimation methods in the textbook, Teacher D would ask students to observe the numbers, to see if there are whole thousands or ten thousands after computation for more convenient computation. If there are issues that do not affect computation results in the calculations could be ignored. In instruction, Teacher D is inclined toward listing various solution methods, because there are differences between the abilities of children. For the computational estimation problems that require preexisting knowledge, the teacher would attempt to allow students to discover the simplicity and uses of computational estimation. They could think about which methods are better and faster, and students are encouraged to flexibly use mathematics. Teacher D believes that whatever implicates computation involves the concept of computational estimation, even using computational estimation values as reference values to compares sizes, or use steps and arm lengths to estimate the length of the classroom; these are all computational estimation abilities. Teacher D also asks students to use computational estimations as much as possible to check answers.

Teacher E

In the computational estimation unit, Teacher E uses the buying and selling with money to reinforce the learning motivation of students. It similarly has the context of number places, but using this as a guide could better inspire student thinking; after all, students have a great sensitivity for money.

Teacher E believes that in computational estimation instruction, the concept of place values have to be very clear for students, and this mathematical concept is very beneficial for computational estimation; they need to observe the correlations among number places, in turn using place values for computational estimation. However, children who are less capable may still use preexisting experiences for computations. Now, such students would be asked to compare with the methods learned in the new unit, to understand the meaning.

In the example question with decimal points: “When Zhijie used a calculator to calculate 210.16×28.7, he forgot to press the decimal point, and got 21016×287=6031592, what should the correct answer be?” Teacher TE stated that it is possible to directly look at the place number to place the decimal point, but it is also possible to use the computational estimation method to make students understand the position of the decimal point, and apply computational estimation to other mathematical concepts.

Teacher E states that the computation of numbers or comparison of the size of numbers could be estimated, and each unit has significouldt requirements for students to check answers with computational estimation, so students should not be unfamiliar with them. This is sort of similar to examination techniques. After all, there are many test questions, and there is no time to recompute each question for verification. Thus, the computational estimation method is used to test whether one is producing outlandish answers. In fact, computational estimation does not have fixed question forms, and have a greater probability of showing up in daily life, with greater changes; this is a very applicable type of mathematics.

Teacher F

After students have preliminary understanding of computational estimation, Teacher F asks students to pose their own examples of when they would use computational estimation, and discuss them. Teacher F points out that when numbers are larger, computational estimation is more frequently used, and students learn which methods could allow them to conduct preliminary computational estimation of the size of number places for easier computations. Of course, this ability should be used in daily life.

In instruction, Teacher F would list the possible solution methods by students. Regardless of whether they are regular computational estimations, students could also solve problems using preexisting experiences. Even though instruction is on the computational estimation unit, if there are other non-computational estimation methods, even if more precise, it is acceptable. At the right time, the teacher would bring in the concept of size comparisons by computational estimation, and it is not necessary to calculate the precise values. For the same context, if there are numbers with greater differences, in the question on the decimal point: “When Zhijie used a calculator to calculate 210.16×28.7, he forgot to press the decimal point, and got 21016×287=6031592, what should the correct answer be?” Teacher F thinks that it is also possible to use computational estimation to let students understand the position of the decimal point, and apply computational estimation to other mathematical concepts. Teacher F believes that computational estimation includes a broad range of concepts, including four arithmetic operations that involve numbers, compare sizes, or even quantity sense, including length, distance, weight, and volume. If these are not calculated precisely, the computational estimation value is taken. Thus, as much as possible, students would be asked to have extension ideas and use computational estimation to check for the reasonable scope of answers. Thus, student life experiences are used to design the problems, with greater sensitivity and variability.

In table 3, the teacher understanding for computational estimation instruction shows that for the six teachers in computational estimation instruction, the three teachers with mathematics and science majors are better able to incorporate computational estimation into the mathematics course or assist computational estimation with other mathematical concepts, combine with computational estimation, and they would also ask students to use computational estimation to verify the reasonableness of answers. They believe that instruction by introduction through life contexts could reinforce student learning motivation, so they could accept open-ended questions, and could accept students to use preexisting experiences to solve problems, and using their subsequent understanding to learn computational estimation; the other three teachers without mathematics and science majors are less proficient at introducing life contexts, incorporating computational estimation into courses and combining with other mathematical concepts, and there are even situations of precise computation instruction. However, with the influence of number of years in teaching, teachers would have greater ability in integrating and flexibly using these mathematical concepts.

Table 3 The understanding of computational estimation instruction by teachers

Computational estimation instruction \ Teacher code	A	B	C	D	E	F
Introduction through life contexts	✓		✓	✓	✓	✓
Ask students to use computational estimation as a checking device	✓	✓		✓	✓	✓
Incorporate computational estimation into the mathematics course	✓			✓	✓	✓
Accept open-ended questions	✓		✓	✓	✓	✓
Preexisting experiences			✓	✓	✓	✓
Regular computational estimation solutions	✓	✓	✓	✓	✓	✓
Compare sizes		✓				✓
Other mathematics concepts			✓		✓	✓
Discard number places depending on the situation			✓	✓	✓	✓

Computational estimation problem-solving by teachers

Computational estimation problem-solving are the actual method used by teachers when they face computational estimation questions when they are not teaching. Summary of the computational estimation problem-solving strategies used by the teachers are as follows:

Teacher A

Teacher TA uses four strategies: rounding strategy, compatible number strategy, special numbers , and fractions.

Below are some of the examples listed by teachers:

TA: Find the multiples relationship, so take 6400, divide it by 80, and then multiply it by 8, about 640.

TA: 32.28 is $32\frac{1}{4}$, 0.75 is $\frac{3}{4}$, and if they are multiplied it is about 24 or 25 dollars! I would turn 0.28 into $\frac{1}{4}$, then $32\frac{1}{4}$ multiplied by $\frac{3}{4}$, so it is 24 plus $\frac{3}{16}$, then $\frac{3}{16}$ is less than half of $\frac{8}{16}$, so it is approximately 24.

Teacher B

Teacher TB uses three strategies: front-end strategy, rounding strategy, compatible number strategy, special numbers s, using fractions, using known or nicer numbers , and distributive property. Among the six teachers, only Teacher TB uses distributive property, and uses the fewest number of strategies among the six teachers.

TB: I would think of 12 multiplied by 11, and multiplied by 11 seems to be slower, and it equals multiplying by (10+1), 11 as 10 then results in 120 as the product, and 1 multiplied by 12 is just 12! It is about 132, and the decimal point is not calculated.

Teacher C

Teacher TC uses five strategies: rounding strategy, compatible number strategy, special numbers s, using fractions, using known or nicer numbers .

TC: First, divide 6470 by 73, and estimate an approximate quotient, it is about 6300 divided by 70, so it is less than 90, then multiplied by 8, which is about 700.

Teacher D

Teacher TD uses four strategies: compatible number strategy, special numbers , using fractions, using known or nicer numbers . Among the six teachers, only Teacher TD does not use the rounding strategy method.

TD: This should be about 140! 13 multiplied by 11 is 143, and this is familiar multiplication.

TD: First calculate $\frac{8}{73}$ is 9, since 73 is close to 72, so calculate $\frac{8}{72}$ as 9, then divide 6470 by 9. Then the one-place division is pretty easy.

Teacher E

Teacher TE uses four strategies: rounding strategy, special numbers , using fractions, using known or nicer numbers .

TE: 0.5. I could find that $\frac{16}{15}$ is close to 1, and $\frac{23}{47}$ is close to 0.5.

TE: I see 0.75 as $\frac{3}{4}$, I quite like using fractions, then $\frac{3}{4}$ could be reduced with 32.28! So it is 8.07, then multiplied by 3.

Teacher F

Teacher TF uses five strategies: front-end strategy, rounding strategy, special numbers, using fractions, using known or nicer numbers . Among the six teachers, only Teacher TF uses the front-end strategy.

TF: Take the high place number, because there are no options! What is it approximately, maybe just 53000＋8000＋2000!

TF: $\frac{16}{15}$ is close to 1, $\frac{23}{47}$ is close to half, so it is about 0.5.

Table 3 Computational estimation problem-solving by teachers

Teacher code / Computational estimation problem-solving	A	B	C	D	E	F
Front-end strategy						✓
Rounding strategy	✓	✓	✓		✓	✓
Compatible number strategy	✓		✓	✓		
Special numbers	✓	✓	✓	✓	✓	✓
Using fractions	✓		✓	✓	✓	✓
Using known or nicer numbers			✓	✓	✓	✓
Distributive property		✓				
Computational estimation strategy types	4	3	5	4	4	5

The above table shows that the six teachers all use comprehensible computational estimation strategies to solve problems, and are flexible in their usage of strategies. Most teachers use the strategies of rounding strategy, unique number places, and fractions. Among them, Teacher TB only uses 3 strategies, the fewest, and needs to increase the flexibility in using strategies. Teachers with mathematics and science majors and teachers who are interested in mathematics but do not have mathematics and science majors are more flexible in their strategies, and the number of years in teaching would also affect their flexibility in using strategies. However, teachers who are non- mathematics and science majors and only have a neutral preference for mathematics have less diverse application of strategies.

CONCLUSIONS AND IMPLICATIONS

Conclusions

Teacher A, B, and C have non-mathematics and science majors, while Teacher D, E, F have mathematics and science majors. Teacher C has been teaching for the longest time, and Teacher D has been teaching for the shortest time, and the other teachers have similar numbers of years in teaching. Teachers with mathematics and science majors generally have taught fewer years than those with non-mathematics and science majors. Teachers with mathematics and science majors are all very interested in mathematics. Teacher A and C both quite like mathematics, while Teacher B has a neutral preference for mathematics.

In computational estimation content, it is found that teachers with mathematics and science majors generally use definitions to interpret computational estimation, while teachers that do not have mathematics and science majors but are interested in mathematics would use life applications to explain the content of computational estimation. Only Teacher D simultaneously uses definitions and life applications for explanations.

In computational estimation instruction, it is found that teachers with mathematics and science majors are better at incorporating computational estimation into the mathematics course, or other mathematical concepts to assist in computational estimation and combine with computational estimation, and they would also ask students to use computational estimation to verify the reasonableness of answers. They believe that instruction by introduction through life contexts could reinforce student learning motivation, so that they could accept open-ended questions. The teachers could also allow students to solve problems from preexisting experiences, and make comparisons to learn computational estimation. Teachers without mathematics and science majors are less adept at introducing life contexts, incorporating computational estimation into curricula, and combining with other mathematical concepts. There are also situations of precise computation instructions. However, with the influence of years of instruction, teachers would become better at integrating and flexible using mathematical concepts. On the whole, teachers with mathematics and science majors and are also interested in mathematics have the richest knowledge of computational estimation instruction, teachers without mathematics and science majors but are interested in mathematics have pretty good computational estimation instruction as well, and teachers without mathematics and science majors or interest in mathematics have weaker computational estimation instruction. In computational estimation problem-solving, it could be found that teachers with mathematics and science majors and teachers without mathematics and science majors but are interested in mathematics are more flexible in using strategies. The length of instruction years would also affect the flexibility of teacher strategies. However, teachers without mathematics and science majors or interest for mathematics have less diverse application of strategies.

Implications

The diversity of teachers' computational estimation instruction and solution strategies do indeed help in elevating student computational estimation abilities, but teachers must first reinforce their personal knowledge in computational estimation instruction. Teachers should avoid only using a few types of computational estimation strategies, and should use different solution strategies based on different contexts to stimulate the flexible thinking of students.

Only teachers who have computational estimation knowledge and other relevant knowledge could stimulate and guide students in improvement in computational estimation.

At the present time, mathematics curricula still focus on obtaining the standard correct answers. Thus, it is suggested that when teachers teach, they could encourage students to make rough computational estimations and guesses for the mathematics questions and determine the reasonable range of the answer before computations to check for the reasonableness of the calculated answers. Teachers should encourage students to use computational estimation in daily life and mathematics solutions. Multilateral application of computational estimation allows students to have more understanding for computational estimation, and increase the tolerance for question errors.

Elementary school teachers are responsible for constructing the foundation of number sense in youngsters, and so it is recommended that teacher-training programs include an emphasis on number sense to ensure the development of dynamic, productive computation and estimation skills in students. Thus, educational administrative institutions should provide mathematics professional development opportunities for in-service teachers who are non-mathematics and science majors, in order to enhance their understanding for computational estimation to effectively incorporate number sense into instruction in mathematics, engaging in meaningful mathematics instruction, and elevating student mathematical abilities.

The reseracher believe that a continuing, consistent emphasis by mathematics teacher educators on estimation, and reasonableness of solutions in number systems courses likely will increase preservice teachers' number sense and will create heightened awareness of computational estimation's usefulness.

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